THE ANALYSIS OF THE DYNAMIC EFFECTS IN VIBRATING AND PULSE PLATE EXTRACTION COLUMNS

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The analysis is given of the forces and pressures acting on various parts of the vibrating and pulse plate extraction column. The relationships are presented for both the instantaneous values of the forces acting on the plate and the bottom of the column, and for the instantaneous and average power input. The results of the analysis are interpreted from the point of view of measurement of the dynamic effect and power input by two methods: The measurement of pressure pulsations at the bottom of the column, and the measurement of forces on the shaft carrying the plates. The vibrating and pulse columns are compared from the dynamic point of view.

The counter-current extractors utilizing periodic motion of the liquid with respect to the packing or plates for intensification of mass transfer have been applied ever more extensively recently. The relative periodic motion can be induced either by pulsation of the liquid in the column, while the plates (or packing) are kept fixed, or by vibration of the plates (or packing). The former type is termed pulse extractors and the latter vibrating extractors. The vibrating and pulse plate extractors considered in this paper are the perforated-plate columns. For the design of these apparatuses it is essential that the information about the acting forces as well as the required power input be available. These data are important not only for proper choice of the motor and the design of individual parts but also for understanding of the processes taking place in the extractor.

Several papers[1–4] dealing with the dynamic effects in pulse extractors have been published. A majority of them is concerned about the particular type of pulsation by air. In such a case the resultant displacement of liquid in the column is given by combined action of the volume and surface forces including periodic variations of pressure of the air cushion causing the motion of liquid. The theoretical and experimental case of pulsation of a liquid in a column by means of a piston performing harmonic motion was investigated in the paper of Jealous and Johnson[1]. Certain simplifying assumptions are typical for all papers cited and bring about discrepancies between the experimental and calculated values of pressure in the column: *1.* The friction resistance in the openings of the plate can be expressed from the orifice equation with constant coefficient of resistance over the whole cycle; *2.* only the acceleration in free cross-section of the column and the pulsator is considered in the calculation of inertia forces, whereas that of the liquid within the openings and their vicinity is neglected.

Earlier papers investigated only the pressure exerted on the bottom, eventually the pressure on a certain part of the wall, but not the pressure acting on the plates. In this paper attention is paid to the forces acting on the plates not only because of their importance for design considerations but also because this paper deals with both the pulse and vibrating plate columns. While the power input of a pulse column is given by the product of the force acting on the piston and its velocity (and hence by the pressure near the bottom of the column or the pulsator arm), the power input of a vibrating column is determined by the forces exerted by the plate on the liquid and by the speed of motion of the plates.

THE OVER-ALL MOMENTUM BALANCE

The relation between the instantaneous force acting on a set of plates and the instantaneous pressure near the bottom of an extractor can be obtained from the momentum balance. Let us consider the case when a liquid phase is passing through the column and its flow is generally unsteady. The plates, mounted on a common shaft, move too and their motion is also unsteady. The macroscopic momentum balance in a closed system containing a liquid and a solid phase can be written as follows:

$$\mathrm{d}\boldsymbol{P}/\mathrm{d}t = \mathrm{d}(\boldsymbol{P}_\mathrm{L} + \boldsymbol{P}_\mathrm{S})/\mathrm{d}t = \sum_\mathrm{i} \boldsymbol{F}_{\mathrm{Ai}} + \boldsymbol{F}_{\mathrm{ML}} + \boldsymbol{F}_{\mathrm{MS}} \,. \qquad (1)$$

The check system considered is sketched in Fig. 1. It consists of the liquid filling that part of the column where the plates are situated, the plates themselves and the shaft. Let the flow of the liquid through the surface *1* and *2* be a piston-type flow. In view of incompressibility and constant cross-section of the column the instantaneous velocities of liquid on surfaces *1* and *2* are equal. Consequently, the system can be defined as one whose boundaries *1* and *2* move with the velocity $\boldsymbol{w}_\mathrm{L}(t)$ and therefore closed. Thus the forces exerted by liquid on the plates are inner ones and as such do not participate in the balance. The accumulation terms on the left hand side of Eq. (*1*) can be expressed by means of the mass of liquid, m_L, and that of the set of plates and the shaft, m_S

$$\boldsymbol{P}_\mathrm{L} = \int_{m_\mathrm{L}} \boldsymbol{u} \, \mathrm{d}m_\mathrm{L} = m_\mathrm{L} \boldsymbol{w}_\mathrm{L} \quad \text{and} \quad \boldsymbol{P}_\mathrm{S} = m_\mathrm{S} \boldsymbol{w}_\mathrm{S} \qquad (2)$$

and

$$\boldsymbol{P}_\mathrm{S} = m_\mathrm{S} \boldsymbol{w}_\mathrm{S} \,. \qquad (3)$$

The surface forces acting on the system can be decomposed into their tangential and normal components

$$\sum_\mathrm{i} \boldsymbol{F}_{\mathrm{Ai}} = \int_\mathrm{A} (\tau \boldsymbol{u} + p\delta) \, \mathrm{d}\boldsymbol{A} + \boldsymbol{F}_\mathrm{R} \,. \qquad (4)$$

$\boldsymbol{F}_R$ is the force exerted by the vibrator on the plates. This force is also transferred on the liquid unless there is friction between the plates and the wall and if the friction in the bearings guiding the shaft is negligible. Since the velocities w_L encountered in extractors are small and the viscosity of continuous phase is also small, the first term to be integrated can be neglected. The net force on the wall of the column is zero owing to the axial symmetry of the system. This is true even though we are dealing with a turbulent system and instantaneous values as long as the scale of turbulence is much smaller than the length of the column. Thus only the forces acting on surfaces *1* and *2* remain. Since in accord with our assumption the flow across these surface is a piston-type flow we can write

$$\sum_i \boldsymbol{F}_{Ai} = (p_1 - p_2)\, \boldsymbol{A}_c + \boldsymbol{F}_R\,, \tag{5}$$

where the vector $\boldsymbol{A}_c$ is parallel to the x axis. Finally, the volume forces are

$$\boldsymbol{F}_{ML} + \boldsymbol{F}_{MS} = (m_L + m_S)\, \boldsymbol{g}\,. \tag{6}$$

As all terms in Eq.(*1*) are vectors parallel to the axis of the column, the relation can be rewritten into the scalar form. With respect to Eq.(*2*) and (*6*)

$$m_L(\mathrm{d}w_L/\mathrm{d}t) + m_S(\mathrm{d}w_S/\mathrm{d}t) = (p_1 - p_2)\, A_c + F_R + (m_L + m_S)\, g\,. \tag{7}$$

In a particular case of a vibrating column w_L = const.,

$$F_{Rv} = (p_{2v} - p_1)\, A_c + m_S(\mathrm{d}w_S/\mathrm{d}t) - (m_L + m_S)\, g\,. \tag{8}$$

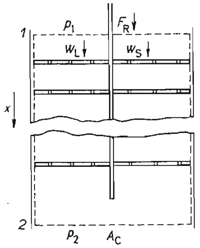

FIG. 1
Check System for Momentum Balance

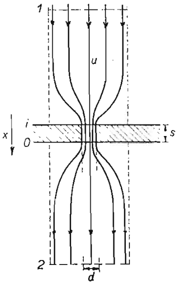

FIG. 2
Check System for Energy Balance

In a particular case of a pulse column $w_S = 0$,

$$F_{Rp} = (p_{2p} - p_1) A_c + m_L(dw_L/dt) - (m_L + m_S) g \,. \tag{9}$$

Eqs (*8*) and (*9*) enable the forces acting on the plates to be calculated from measurements of the pressure, p_2, near the bottom of the column which is easier. The force acting on the plates of vibrating column is

$$F_{Pv} = F_{Rv} + m_S(1 - \varrho_L/\varrho_S) g - m_S(dw_S/dt) \tag{10}$$

and for a pulse column

$$F_{Pp} = F_{Rp} + m_S(1 - \varrho_L/\varrho_S) g \,. \tag{10a}$$

In a pulse column F_{Rp} represents a force acting on the joint of the plates regardles of construction details. The group ϱ_L/ϱ_S represents the buoyancy force which, as an inner force, does not appear in the over-all balance. The instantaneous power input supplied to the liquid in a vibrating extractor is obtained as a product of forces exerted on liquid by the plates and the instantaneous velocity of the plates. Thus with respect to Eq. (*8*)

$$N_{Lv} = w_S[(p_{2v} - p_1) A_c - (m_L + m_S\varrho_L/\varrho_S) g] \,. \tag{11}$$

The instantaneous power input supplied to the liquid in a pulse extractor is a product of the force near the bottom and the velocity of liquid:

$$N_{Lp} = - w_L p_{2p} A_c \,. \tag{12}$$

The velocity and acceleration are considered continuous in time. A comparison of instantaneous power inputs in vibrating and pulse extractors can be made at the same relative velocity with respect to the plates $w_S = - w_L$. Since the pressures p_2 are different in both columns they have to be substituted with the aid of Eqs (*8*)–(*10*) by forces acting on the plates, which are equal at the same relative velocity of liquid with respect to the plates.

$$N_{Lp} - N_{Lv} = - w_L[p_1 A_c + (m_L + m_S\varrho_L/\varrho_S) g - m_L(dw_L/dt)] \,. \tag{13}$$

The difference between both types follows from the periodic motion of the whole liquid content of the pulse column. The work is done against the outer pressure and the changes in accumulation of the kinetic and potential energy occur. The average values of the power input for the whole cycle, however, are equal in both columns as follows from integration of Eq. (*13*):

$$\overline{N_{Lp} - N_{Lv}} = \frac{1}{T} \{[p_1 A_C + (m_L + m_S \varrho_L/\varrho_S) g] \int_0^T - w_L \, dt + (m_L/2) \int_0^T (dw^2/dt) \, dt\} ;$$

$$\int_0^T - w_L \, dt = - \oint d\sigma = 0 ; \quad \int_0^T (dw_L^2/dt) \, dt = \oint dw^2 = 0 ;$$

$$\overline{N}_{Lp} = \overline{N}_{Lv} . \tag{14}$$

To compare the strain of the driving mechanism, one has to compare the power input on the shaft of a vibrating extractor, N_{Rv}, and on the piston of a pulsator, N_{Rp}

$$N_{Rv} = w_S F_{Rv} = w_S[(p_{2v} - p_1) A_c + m_S(dw_S/dt) - (m_L + m_S) g] , \tag{15}$$

$$N_{Rp} = - w_L[p_{2p} A_c - m_l(A_c/A_l)^2 (dw_L/dt)] . \tag{16}$$

m_l stands for the mass of liquid in the pulsator arm and A_l is its cross-section. The friction in the pulsator arm is neglected; the arm is horizontal. By analogous arrangements as those used in deriving Eq. (*13*) we get

$$N_{Rp} - N_{Rv} = - w_L\{p_1 A_c + (m_L + m_S) g - [m_L + m_{Ll}(A_c/A_l) - m_S] (dw_L/dt)\} . \tag{17}$$

Although the average values of the power input are again equal, it is the maximum values, not the average ones, that are important for dimensioning of the motor unless a flywheel is used. Considering that m_L is usually several times greater than m_S, Eq. (*17*) shows clearly the increase of the instantaneous power input and the strain of the driving mechanism of a pulse column in comparison with that of the vibrating plate column with increasing volume of the column, and velocity and acceleration of pulsation. Increasing pressure on the liquid level has also an unfavourable effect on the function of the pulsator. From comparison of a vibrating plate and pulse extractors at the same relative velocity of liquid with respect to the plates, and hence at the same F_p, it further follows:

$$p_{2p} - p_{2v} = - (m_L/A_c) (dw_L/dt) . \tag{18}$$

All preceding relations hold regardless of the form of functions $w_S(t)$ and $w_L(t)$. A stipulation for Eq. (*14*) is that these functions be periodic. For Eqs (*13*), (*14*), (*17*) and (*18*) it is further assumed that $w_S = - w_L$, *i.e.* the average velocity of liquid $\bar{w}_L = (1/T) \int_0^T w_L \, dt$ is neglected. Let us denote

$$w_L(t) = w'_L(t) + \bar{w}_L .$$

For comparison of the vibrating and pulse plate columns one can then put in Eqs (*13*) and (*14*) more generally $-w_S(t) + w_L = w'_L(t) + w_L$. Thus these equations remain valid, except that w_L is replaced by w'_L. More accurately, w'_L should appear in Eq. (*16*) as well as in Eq. (*17*), which then remains valid. All quantities F_R, F_P, p_2 and N can be decomposed into the average value and the pulsation component. While for the velocity one can arrive at w_L by superimposing the pulsation w'_L on the mean velocity $\bar{w}_L$ (as we are dealing with velocities averaged over the cross-section), the components of the forces, pressures and power inputs $\bar{F}$, $\bar{p}_2$, $\bar{N}$ are not generally identical with the values of these quantities at the stationary flow through the column at $\bar{w}_L$. The reason is that these quantities, as will be shown below, depend on the velocity field in the column and hence both their average and pulsation components are functions of the average and pulsation component of velocity.

Energy Balance

To obtain a relation for the instantaneous values of p_2, or F_R, we set up first the energy balance for a liquid system containing one immobile perforated plate with uniformly spaced openings of circular cross-section. The plate itself has a finite thickness. No interference is assumed between the flows through the individual openings. Hence, for the check volume it suffices to take a tube around one opening coaxial with its axis and such that all liquid passing through the opening remains within the tube (Fig. 2). For the time being let us assume an ideal liquid and a potential flow. The equation of motion is:

$$\partial \boldsymbol{u}/\partial t + \nabla(\boldsymbol{u}^2/2) = -\boldsymbol{g} + \nabla p/\varrho_L . \qquad (19)$$

For the streamline passing through the center of the opening we get by integration between x_1 and x_2:

$$p_1/\varrho_L + u_1^2/2 - \boldsymbol{g}x_1 = p_2/\varrho_L + u_2^2/2 - \boldsymbol{g}x_2 + \int_{x_1}^{x_2} (\partial u/\partial t)\, \mathrm{d}x . \qquad (20)$$

The integral on the right hand side represents the effect of unsteadiness of the flow. Eq. (*20*) can be extended to real fluids by introducing the losses, f_{12}. Further we assume that the velocity profile in cross-sections *1* and *2* is flat and, accordingly, $u_1 = u_2 = w_L$

$$(\pi_1 - \pi_2)/\varrho = \int_{x_1}^{x_2} (\partial u/\partial t)\, \mathrm{d}x + f_{12} , \qquad (21)$$

where $\pi = p - \varrho \boldsymbol{g} x$.

FRICTION LOSSES

The derivation of the term for friction losses is based on the assumption that at moderate accelerations the losses under unsteady flow would not differ appreciably from those under the steady flow. The total losses can be divided into the losses at the inlet of the opening, f_{1i}, within the opening, f_{i0}, and at the exit, f_{02}:

$$f_{12} = f_{1i} + f_{i0} + f_{02}\,. \tag{22}$$

The inlet losses, f_{1i}, are important only for small values of plate free cross-section, ε, and large values of $\mathrm{Re} = dw_L/\nu\varepsilon$. Then we have approximately[5]

$$f_{1i} = 0{\cdot}5(1 - \varepsilon)\,. \tag{23}$$

The losses in the opening must be expressed for two cases: The laminar and the turbulent flow. The problem is to assess the losses in a short tube of circular cross-section with sharp leading and trailing edges. This case has been examined in the preceding paper[6]. For the laminar flow, Schiller's model[7] has been verified which attributes the losses to the changes of momentum of the potential flow within the core at gradual formation of the laminar boundary layer on the walls. The resulting relations are as follows

$$f_{i0} = (\Phi^2 - 1)\,(w_L|w_L|/\varepsilon^2)\,, \tag{24}$$

$$\Phi = \Phi(s/d\ \mathrm{Re}) = u_0\varepsilon/w_L\,, \tag{24a}$$

where u_0 is the velocity on the axis at the outlet.

$$\begin{aligned} s/d\ \mathrm{Re} = {} & (1/240)\,\{58(\Phi - 1) - 66\ln\Phi - 17\sqrt{(2)}\,(3\Phi - \Phi^2)^{0{\cdot}5} - \\ & - 48\sqrt{(2)}\,[(3 - \Phi)/\Phi]^{0{\cdot}5} + 130 - 63\sqrt{(2)}\arcsin(\Phi/3)^{0{\cdot}5} + \\ & + 63\sqrt{(2)}\arcsin(1/3)^{0{\cdot}5} - (48/\sqrt{(2)})\arcsin[(2\Phi)/3 - 1] - \\ & - (48/\sqrt{(2)}\arcsin(1/3)\}\,. \end{aligned} \tag{24b}$$

For the losses under the turbulent flow in the opening one can use Blasius' relation correlating the losses in a straight tube with the correction on contraction of the jet

$$f_{i0} = [(1 + \beta)^2/\beta^2 + 0{\cdot}316s/(d\ \mathrm{Re})^{1/4}]\,(w_L|w_L|/2\varepsilon^2)\,, \tag{25}$$

where β is the coefficient of contraction.

A relation for the losses by sudden expansion can be derived in a familiar way from the momentum and energy balances in section $0-2$. Under the laminar regime in the opening one has to consider the velocity profile in the exit cross-section *0*. The correction coefficients for the momentum, α_M, and energy, α_E: are defined as

$$\alpha_M = \varepsilon^2 \int_0^r 2yu(y)^2 \, \mathrm{d}y / r^2 w_L^2 \; ; \; \alpha_E = \varepsilon^3 \int_0^r 2yu(y)^3 \, \mathrm{d}y / r^2 w_L^3 \, . \tag{26}$$

Then:

$$f_{02} = (\varepsilon^2 - 2\alpha_M \varepsilon + \alpha_E) \, w_L |w_L| / 2\varepsilon^2 \, . \tag{27}$$

The total losses according to Eqs (*22*) through (*27*) for the laminar flow in the opening are then

$$f_{12} = (\varepsilon^2 - 2\alpha_M \varepsilon + \alpha_E - 1 + \Phi^2) \, (w_L |w_L| / 2\varepsilon^2) \tag{28}$$

and for the turbulent flow in the opening

$$f_{12} = \left[0{\cdot}5(1 - \varepsilon) + \varepsilon^2 - 2\varepsilon + 1 + (1 - \beta)^2/\beta^2 + 0{\cdot}316 s/(d \, \mathrm{Re})^{1/4}\right] . \\ . \, (w_L |w_L| / 2\varepsilon^2) \, . \tag{29}$$

Unsteady Flow Contribution

The velocity field in the neighborhood of the plate is too complex. Thus a realistic expression of the integral on the right hand side of Eq. (*21*) is not easy. We shall therefore introduce an empirical quantity, the equivalent thickness of the plate, s_E. This quantity is defined as the height of the liquid column moving at the velocity w_L/ε and causing an increase of the inertia term, when compared with its value in an empty column, equivalent to the effect due to the presence of the plate.

$$[(1 - \varepsilon) \, s_E/\varepsilon] \, (\mathrm{d}w_L/\mathrm{d}t) = \int_{x_1}^{x_2} (\partial(u - w_L)/\partial t) \, \mathrm{d}x \, ,$$

$$\int_{x_1}^{x_2} (\partial u/\partial t) \, \mathrm{d}x = [x_2 - x_1 + (1 - \varepsilon) \, s_E/\varepsilon] \, (\mathrm{d}w_L/\mathrm{d}t) \, . \tag{30}$$

RESULTS AND DISCUSSION

At sufficient spacing of the plates one can assume that their effects are additive. Thus for instantaneous pressure p_{2p} near the bottom of an n-plate pulse column we get from Eqs (*21*) and (*30*)

$$p_{2p} - p_1 = \varrho_L \{ gL - [L + n(1 - \varepsilon) \, s_E/\varepsilon] \, (\mathrm{d}w_L/\mathrm{d}t) - n f_{12} \} \, , \tag{31}$$

where for expressing f_{12} we have Eq. (*28*) or (*29*).

The instantaneous force acting on the shaft can be easily calculated from these relationships, Eq. (*8*) and considering further Eq. (*18*) and the condition $- w_S + \bar{w}_L = w_L$.

From Eqs (*31*) and (*18*) and with respect to $m_L/A_C = \varrho L$ and $dw_L/dt = - dw_S/dt$, one obtains

$$p_{2v} - p_1 = \varrho_L\{gL + [n(1 - \varepsilon) s_E/\varepsilon] (dw_S/dt) - nf_{12}\} . \qquad (31a)$$

Note that $- w_S + \bar{w}_L = w_L$ has to be substituted in expression for f_{12}. On substitution in Eq. (*8*) we obtain finally

$$F_{R_v} = - m_S(1 - \varrho_L/\varrho_S) g + [m_S + \varrho_L A_C n(1 - \varepsilon) s_E/\varepsilon] (dw_S/dt) - n\varrho_L A_C f_{12} . \qquad (32)$$

The correction for the buoyancy in the first term on the right hand side of Eq. (*32*) appeared as a difference of m_L and $\varrho_L A_C L$.

The relations (*31*) and (*32*) contain an unknown empirical quantity, S_E, which may be a function of the amplitude and the frequency (a, f), the form of oscillations, the diameter of the plate openings (d), the plate free cross-section (ε), the thickness of the plate (s), the viscosity and density of liquid phase (ν, ϱ_L) and the average velocity $\bar{w}_L$. On considering only harmonic oscillations one can assume following dependence between dimensionless groups

$$\bar{s}_E/s = \varrho(afd/\nu, \bar{w}_L d/\nu, a/s, d/s, \varepsilon) \qquad (33)$$

the form of which is yet to be found experimentally. The bar over s_E designates the average value over the whole cycle. The results of experiments which are now conducted suggest that the shapes of the experimental and theoretical dependences $F_R(t)$ and $p_2(t)$ are in good agreement. This indicates not only that the average values of s_E can be used for characterizing the inertia forces, but also that the friction losses at a steady flow can be used for description of an unsteady situation within the given range of variables. The measured values of F_R and p_2 also satisfy Eq. (7).

This analysis concerns a single phase flow. The presence of another phase requires corrections of individual terms. Nevertheless, it may be expected that owing to small differences in densities the single phase calculation provides estimates of forces and power input sufficiently accurate for practice. In the part of this paper where the friction losses were formulated it was assumed that the diameter of the openings was close to the thickness of the plate. For certain types of columns (*e.g.*: Karr[8]) with large openings it may be expected that a relationship for losses in an orifice will be more suitable.

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LIST OF SYMBOLS

a amplitude of pulse
$\mathbf{A}$ vector surface
A_C column cross-section
d diameter of plate opening
f losses in energy equation, frequency of pulses in Eq. (*33*)
$\mathbf{F}$ force
g acceleration due to gravity
L length of column
m mass
n number of plates
$\mathbf{n} = d\mathbf{A}/|d\mathbf{A}|$ unit vector of surface
N power input
p static pressure
$\mathbf{P}$ momentum
r radius of plate openings
s thickness of plate
s_E equivalent thickness of plate
t time
T duration of cycle
u velocity on the axis of opening
$u(y)$ velocity profile on exit from opening
$\mathbf{u}$ local instantaneous velocity of liquid
$\mathbf{w}_L$ instantaneous superficial velocity
$\mathbf{w}_S$ instantaneous velocity of plates
α_M, α_E defined by Eq. (*26*)
β coefficient of contraction
δ unit tensor
ε specific free cross-section of plate (porosity)
ν kinematic viscosity
$\pi = p - \varrho_L g$
ϱ density
σ path
τ shear stress
ϕ defined by Eq. (*24a*)
$\mathrm{Re} = dw_L/\nu\varepsilon$ Reynolds number

Subscripts

A surface
L, S liquid or solid phase
M mass
P plate
R shaft
1 pulsating arm
p pulsation column
v vibration column

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